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KINKING OF AN INTERFACE CRACK BETWEEN TWO DISSIMILAR ANISOTROPIC ELASTIC SOLIDS

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Abstract—A detailed analysis of kinking of an interface crack between two dissimilar anisotropic elastic solids is presented in this paper. The branched crack is considered as a distributed dislocation.

A set of the singular integral equations for the distribution function of the dislocation density is developed.

Explicit formulas of the stress intensity factors and the energy release rates for the branched crack are given for orthotropic bimaterials and misoriented orthotropic bicrystals.

The role of the stress parallel to the interface, σ_0 is taken into account in these formulas.

The interface crack can advance either by continued extension along the interface or by kinking out of the interface into one of the adjoining materials. This competition depends on the ratio of the energy release rates for interface cracking and for kinking out of the interface and the ratio of interface toughness to substrate toughness.

Throughout the paper, the influences of the inplane stress σ_0 on the stress intensity factors and the energy release rates for the branched crack, which can significantly alter the conditions for interface cracking, are emphasized.

1. INTRODUCTION

The interface fracture mechanics for two dissimilar anisotropic materials has attracted many scientists' attention.

Gotoh (1967) analysed the interface crack problem for the anisotropic plate based on the Dugdale mode. Clements (1971) presented the general formulas for the interface crack between two dissimilar anisotropic solids using the Stroh's (1958) theory of anisotropic elastic media. He introduced the six piecewise analytical potentials and established the well-known Hilbert problem.

Several basic problems have been solved by Wills (1971), Ting (1986), Wang and Choi (1983), Bassani and Qu (1989), Qu and Bassani (1989), Wu (1990), Gao *et al.* (1992) among others. A significant progress has been made by Suo (1990). He established the general singular fields for the interface crack between two dissimilar anisotropic media using the complex function vector.

The kinking of a crack out of an interface between two dissimilar isotropic materials is solved by He and Hutchinson (1989), He *et al.* (1991) for the case of semi-infinite cracks.

Using the formulation in Clements (1971) and Wang and Choi (1983), and the Green's function, a solution has been presented by Miller and Stock (1989) for the problem of a crack branching off the interface between two dissimilar anisotropic materials. Numerical results for the stress intensity factor of the branch crack are obtained for some special cases in their paper.

Wang *et al.* (1992) developed the concepts of mode mixity and toughness surface for an interface crack in anisotropic solids. Explicit formulae for stress intensity factors and energy release rates for branch crack are presented in their paper. Many typical numerical results are obtained.

This paper is a continuous development of the work by Wang *et al.* (1992). The role of the stress σ_0 in the substrate parallel to the interface in kinking of a crack out of an interface between two dissimilar anisotropic materials is investigated in detail.

It has been shown that the influences of the stress σ_0 on the energy release rate of the branch crack are significant when the length a of the branch crack is not very small. A thorough analysis for the energy release rates G^{kink} for the branch crack is presented here.

Explicit formulae for the stress intensity factors and the energy release rates are developed with emphasis on the contribution of the stress σ_0.

2. BASIC FORMULAE

The formulation in Suo (1990), Wang *et al.* (1992) will be used throughout the paper.

The displacements u_i, stresses σ_{2i} and resultant forces f_i on an arc AB can be expressed as:

$$\begin{cases} u_i = 2\mathrm{Re}\sum_{i=1}^{3} A_{ij}\varphi_j(z_j), \\ f_i = -2\mathrm{Re}\sum_{j=1}^{3} L_{ij}\varphi_j(z_j)|_A^B, \\ \sigma_{2i} = 2\mathrm{Re}\sum_{j=1}^{3} L_{ij}\varphi_j'(z_j). \end{cases} \tag{1}$$

Introduce following vector functions:

$$\varphi(z) = [\varphi_1(z_1), \varphi_2(z_2), \varphi_3(z_3)]^T$$
$$\Phi(z) = [\varphi_1(z), \varphi_2(z), \varphi_3(z)]^T.$$

The interaction problem of dislocation versus interface crack in anisotropic media has been solved by Suo (1990).

Consider a dislocation line in the direction perpendicular to x, y plane with Burger's vector $\mathbf{b}$ at the point (x_0, y_0) in material 2, as shown in Fig. 1.

We have:

$$\Phi'(z) = \Phi_0'(z) + C^{(2)}\bar{\Phi}_0'(z) + L_2^{-1}\bar{H}^{-1}Hh(z), \quad z\in\Omega^- \tag{2}$$

where

$$C^{(2)} = L_2^{-1}\bar{H}^{-1}(\bar{B}_2-\bar{B}_1)\bar{L}_2,$$
$$\Phi_0(z) = [\varphi_{01}(z), \varphi_{02}(z), \varphi_{03}(z)]^T$$
$$\varphi_{0j}(z) = \mathrm{d}_j Ln(z-s_j), s_j = x_0+\mu_j y_0,$$
$$d = [d_1, d_2, d_3]^T = L_2^{-1}(B_2+\bar{B}_2)^{-1}\mathbf{b}/2\pi,$$
$$h(z) = h_1(z)W + h_2(z)\bar{W} + h_s(z)W_3.$$

Matrices **A**, **B**, **L**, **H** and vectors **W**, $\bar{\mathbf{W}}$, $\mathbf{W}_3$ are introduced by Suo (1990) and Miller (1989). The subscript 2 indicates quantities for material 2.

For the case of a semi-infinite crack, Wang *et al.* (1992) show that:

$$h_i(z) = -R_i^T\mathbf{b}/2\pi, \quad i = 1, 2, 3, \tag{3}$$

$$R_i^T = [P_i^T D(z, s, \gamma_i)C + Q_i^T D(z, \bar{s}, \gamma_i)\bar{C}](1-\beta_i)/2, \quad i = 1, 2, 3, \tag{4}$$

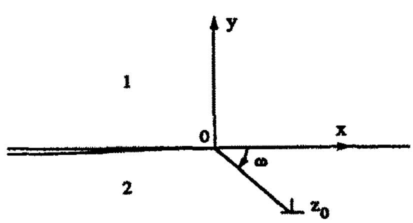

Fig. 1. Interaction between a dislocation line with a traction free crack.

where

$$\beta_1 = \beta, \quad \beta_2 = -\beta, \quad \beta_3 = 0,$$

$$\gamma_1 = \tfrac{1}{2} - i\varepsilon, \quad \gamma_2 = \tfrac{1}{2} + i\varepsilon, \quad \gamma_3 = \tfrac{1}{2},$$

$$\begin{cases} P_1^T = \bar{\mathbf{W}}^T(\mathbf{B}_2+\bar{\mathbf{B}}_2)\mathbf{L}_2/\Delta_0, & Q_1^T = \mathrm{e}^{-2\pi\varepsilon}\bar{\mathbf{W}}^T(\mathbf{B}_2+\bar{\mathbf{B}}_2)\bar{\mathbf{L}}_2/\Delta_0, \\ P_2^T = \bar{Q}_1^T, & Q_2^T = \bar{P}_1^T, \\ P_3^T = \mathbf{W}_3^T(\mathbf{B}_2+\bar{\mathbf{B}}_2)\mathbf{L}_2/\Delta_3, & Q_3^T = \mathbf{W}_3^T(\mathbf{B}_2+\bar{\mathbf{B}}_2)\bar{\mathbf{L}}_2/\Delta_3, \end{cases}$$

$$\Delta_0 = \bar{\mathbf{W}}^T\mathbf{H}\mathbf{W}, \quad \Delta_3 = \mathbf{W}_3^T\mathbf{H}\mathbf{W}_3,$$

$$\varepsilon = \frac{1}{2\pi}\mathbf{L}n\frac{(1-\beta)}{(1+\beta)}, \tag{5}$$

$$D(z,s,\gamma) = \mathrm{diag}\left\{\frac{[1-(z/s_1)^{-\gamma}]}{(z-s_1)}, \frac{[1-(z/s_2)^{-\gamma}]}{(z-s_2)}, \frac{[1-(z/s_3)^{-\gamma}]}{(z-s_3)}\right\}. \tag{6}$$

Introducing the following matrices:

$$\mathbf{T} = \mathbf{L}_2^{-1}\bar{\mathbf{H}}^{-1}\mathbf{H}, \tag{7}$$

$$\mathbf{R} = \mathbf{W}R_1^T + \bar{\mathbf{W}}R_2^T + \mathbf{W}_3R_3^T, \tag{8}$$

$$\mathbf{C} = \mathbf{L}_2^{-1}(\mathbf{B}_2+\bar{\mathbf{B}}_2)^{-1}, \tag{9}$$

we obtained:

$$\varphi_j'(z) = V_{jk}^{(0)}(z,s)b_k/2\pi + V_{jk}(z,s)b_k/2\pi, \tag{10}$$

$$V_{jk}^{(0)}(z,s) = \frac{C_{jk}}{(z-s_j)}, \quad j \text{ no sum}, \tag{11}$$

$$V_{jk}(z,s) = \sum_{m=1}^{3}\frac{C_{jm}^{(2)}\bar{C}_{mk}}{(z-\bar{s}_m)} - \sum_{m=1}^{3}T_{jm}R_{mk}. \tag{12}$$

For the branch crack as shown in Fig. 2, we have:

$$z = t\,\mathrm{e}^{-i\omega}, \quad z_j = t\cdot\omega_j,$$

$$\omega_j = \cos\omega - \mu_j\sin\omega.$$

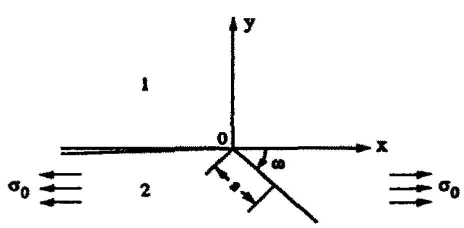

Fig. 2. A branched interface crack.

The traction components p_i on the branch segment, take the forms:

$$p_i = \sigma_{ij} n_j = \sigma_{i1} \sin \omega + \sigma_{i2} \cos \omega$$
$$= 2\mathrm{Re} \sum_{j=1}^{3} L_{ij} \omega_j \varphi_j'(z_j).$$

Noting the branch crack is in the material 2, it follows:

$$\mathbf{L}_2 \mathbf{D}_\omega \varphi'_{\text{total}}(z) + \bar{\mathbf{L}}_2 \bar{\mathbf{D}}_\omega \overline{\varphi'_{\text{total}}(z)} = 0, \tag{13}$$

here

$$\mathbf{D}_\omega = \mathrm{diag}\,[\omega_1, \omega_2, \omega_3]$$
$$\varphi_{\text{total}} = \varphi^{(0)}(z) + \boldsymbol{\varphi}^{(d)}(z). \tag{14}$$

The function vector $\boldsymbol{\varphi}^{(0)}(z)$ is the potential vector prior to kinking. The function vector $\boldsymbol{\varphi}^{(d)}(z)$ is the potential vector due to the distributed dislocation on the line segment of the branch crack. We have:

$$\boldsymbol{\varphi}_i^{(d)'}(z) = \frac{1}{2\pi} \int_0^a \sum_{k=1}^{3} \{ V_{ik}^{(0)}(z,s) b_k(s) + V_{ik}(z,s) b_k(s) \} \,\mathrm{d}\hat{s} \tag{15}$$

where $\hat{s} = |s|$, a is the length of the branch crack.

Substituting eqn (15) into eqn (13), results in:

$$\mathrm{Re} \int_0^a [U_{ik}^{(0)}(z,s) + U_{ik}(z,s)] b_k(s) \,\mathrm{d}\hat{s} = -\pi p_i^{(0)}, \quad z = t\,\mathrm{e}^{-i\omega}, \tag{16}$$

where $p_i^{(0)}$ are the traction components prior to kinking:

$$\begin{cases} U_{ik}^{(0)}(z,s) = \sum_{j=1}^{3} (\mathbf{L}_{ij}\omega_j)_2 V_{jk}^{(0)}(z_j, s) \\ U_{ik}(z,s) = \sum_{j=1}^{3} (\mathbf{L}_{ij}\omega_j)_2 V_{jk}(z_j, s). \end{cases} \tag{17}$$

The eqn (16) is the governing equation for the unknown dislocation density $b(s)$. This is a set of coupled singular integral equations.

The eqn (16) can be represented as:

$$\mathrm{Re} \left\{ \int_0^a \frac{M_{ik} b_k(\tau)\,\mathrm{d}\tau}{(\tau - t)} + 2\int_0^a K_{ik}(\tau, t) b_k(\tau)\, d\tau \right\} = \pi p_i^{(0)}, \quad 0 < t < a, \tag{18}$$

where $\tau = \hat{s} = |s|$, $s = \tau\,\mathrm{e}^{-i\omega}$

$$M = (\mathbf{B}_2 + \bar{\mathbf{B}}_2)^{-1}, \tag{19}$$

$$2K_{ik}(\tau, t) = \sum_{j=1}^{3} (L_{ij}\omega_j)_2 \left\{ \sum_{m=1}^{3} \frac{C_{jm}\bar{C}_{mk}}{(\bar{\omega}_m \tau - \omega_j t)} + \sum_{m=1}^{3} T_{jm} R_{mk}(z_j, s) \right\}, \quad z = t\,\mathrm{e}^{-i\omega}. \tag{20}$$

3. STRESS INTENSITY FACTORS AND ENERGY RELEASE RATE FOR BRANCH CRACK

We consider first the traction components $p_i^{(0)}$. The stress fields near the interface crack prior to kinking are

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}}\{\mathrm{Re}\,(Kr^{i\varepsilon})\tilde{\sigma}_{ij}^{(1)}(\theta)+\mathrm{Im}\,(Kr^{i\varepsilon})\tilde{\sigma}_{ij}^{(2)}(\theta)+K_3\tilde{\sigma}_{ij}^{(3)}(\theta)\}+\sigma_0\delta_{i1}\delta_{j1}, \tag{21}$$

where σ_0 indicates the stress component in material 2 parallel to the interface $\sigma_0 = \sigma_{11}$. As pointed out by Rice and Shih (1965), a jump in the parallel stresses across the interface is required in general. We are only interested in the stress component in material 2 parallel to the interface $\sigma_{11} = \sigma_0$.

The function vector $\varphi^{(0)}(z)$ corresponding to the singularity stress fields is taken in the form:

$$\Phi^{(0)}(z) = \mathbf{L}_2^{-1}\bar{\mathbf{H}}^{-1}\mathbf{H}h^{(0)}(z), \quad z\in\Omega^-, \tag{22}$$

$$\begin{aligned} h^{(0)}(z) &= \frac{e^{\pi\varepsilon}\,Kz^{i\varepsilon}\mathbf{W}+e^{-\pi\varepsilon}\,\bar{K}z^{-i\varepsilon}\bar{\mathbf{W}}}{2\sqrt{2\pi z}\cosh\pi\varepsilon}+\frac{K_3\mathbf{W}_3}{2\sqrt{2\pi z}} \\ &= h_1^{(0)}(z)\mathbf{W}+h_2^{(0)}(z)\bar{\mathbf{W}}+h_3^{(0)}(z)\mathbf{W}_3. \end{aligned} \tag{23}$$

It follows (including the contribution of σ_0):

$$p_i^{(0)} = 2\mathrm{Re}\left\{\sum_{j=1}^{3}(L_{ij}\omega_j)_2T_{ji}^{*}h_j^{(0)}(z_j)\right\}+\delta_{1i}\sigma_0\sin\omega, \tag{24}$$

where

$$\begin{aligned} T^* &= T\cdot[\mathbf{W},\bar{\mathbf{W}},\mathbf{W}_3], \\ h_1^{(0)}(z) &= e^{\pi\varepsilon}Kz^{i\varepsilon}/2\sqrt{2\pi z}\cosh\pi\varepsilon, \\ h_2^{(0)}(z) &= e^{-\pi\varepsilon}\bar{K}z^{-i\varepsilon}/2\sqrt{2\pi z}\cosh\pi\varepsilon, \\ h_3^{(0)}(z) &= K_3/2\sqrt{2\pi z}. \end{aligned}$$

We discuss only the inplane problem.

Following the work of He *et al.* (1991) and Wang *et al.* (1992). The stress intensity factors at the kink tip can be represented as:

$$K_{\mathrm{I}}^{\mathrm{kink}}+iK_{\mathrm{II}}^{\mathrm{kink}} = cKa^{i\varepsilon}+\overline{dKa^{-i\varepsilon}}+Б\,\sigma_0\sqrt{a}, \tag{25}$$

where c, d, Б are the complex coefficients. They are the function of ω and elastic constants of materials 1 and 2.

Equation (25) can be rewritten as:

$$\begin{aligned} K_{\mathrm{I}}^{\mathrm{kink}} &= c_{11}\,\mathrm{Re}\,(Ka^{i\varepsilon})+c_{12}\,\mathrm{Im}\,(Ka^{i\varepsilon})+Б_1\sigma_0\sqrt{a}, \\ K_{\mathrm{II}}^{\mathrm{kink}} &= c_{21}\,\mathrm{Re}\,(Ka^{i\varepsilon})+c_{22}\,\mathrm{Im}\,(Ka^{i\varepsilon})+Б_2\sigma_0\sqrt{a} \end{aligned} \tag{26}$$

here

$$\begin{aligned} c_{11} &= \mathrm{Re}\,(c+d), \quad c_{12} = -\mathrm{Im}\,(c+d), \\ c_{21} &= \mathrm{Im}\,(c-d), \quad c_{22} = \mathrm{Re}\,(c-d). \end{aligned}$$

The coefficients c_{ij} have been given by Wang *et al.* (1992).

The energy release rate of interface crack is:

$$G_i = \bar{\mathbf{W}}^T(\mathbf{H}+\bar{\mathbf{H}})\mathbf{W}K\bar{K}/4\cosh^2\pi\varepsilon. \tag{27}$$

Introduce the equivalent elastic modulus E_* for the interface:

$$\frac{1}{E^*} = \bar{\mathbf{W}}^T(\mathbf{H}+\bar{\mathbf{H}})\mathbf{W}/4\cosh^2\pi\varepsilon.$$

Equation (27) can be rewritten as:

$$G_i = (K_1^2+K_2^2)/E_*. \tag{28}$$

The energy release rate G^{kink} for the branch crack takes the form:

$$G^{\text{kink}} = k_*^T(B+\bar{B})_* k_*/4, \tag{29}$$

where

$$k_* = [K_{\text{II}}^{\text{kink}}, K_{\text{I}}^{\text{kink}}]^T, \tag{30}$$

$$\mathbf{B}_* = R_*\mathbf{B}R_*^T \tag{31}$$

$$R_* = \begin{bmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{bmatrix}. \tag{32}$$

Substitute eqns (30), (31) into eqn (29), we arrive at†

$$\begin{aligned} G^{\text{kink}} &= \tfrac{1}{2}\{(B_{11}\cos^2\omega+B_{22}\sin^2\omega)(K_{\text{II}}^{\text{kink}})^2+2(B_{11}-B_{22})\cos\omega\cdot\sin\omega\, K_{\text{I}}^{\text{kink}}K_{\text{II}}^{\text{kink}} \\ &\quad +(B_{11}\sin^2\omega+B_{22}\cos^2\omega)(K_{\text{I}}^{\text{kink}})^2\} \\ &= \tfrac{1}{2}\{B_{11}(\sin\omega\, K_{\text{I}}^{\text{kink}}+\cos\omega\, K_{\text{II}}^{\text{kink}})^2+B_{22}(\cos\omega\, K_{\text{I}}^{\text{kink}}-\sin\omega\, K_{\text{II}}^{\text{kink}})^2\}. \end{aligned} \tag{33}$$

Using the eqn (26), it results:

$$G^{\text{kink}} = \tfrac{1}{2}\{B_{22}(e_{11}K_1^*+e_{12}K_2^*+\text{Б}_r\sigma_0\sqrt{a})^2+B_{11}(e_{21}K_1^*+e_{22}K_2^*+\text{Б}_\theta\sigma_0\sqrt{a})^2\}, \tag{34}$$

where B_{ij} are the elements of matrix $\mathbf{B}_2$:

$$\begin{cases} e_{11} = \cos\omega\cdot c_{11}-\sin\omega\cdot c_{21}, \\ e_{12} = \cos\omega\cdot c_{12}-\sin\omega\cdot c_{22}, \\ e_{21} = \sin\omega\cdot c_{21}+\cos\omega\cdot c_{21}, \\ e_{22} = \sin\omega\cdot c_{22}+\cos\omega\cdot c_{22}, \end{cases} \tag{35}$$

$$\begin{cases} \text{Б}_r = \cos\omega\cdot\ \text{Б}_1-\sin\omega\cdot\ \text{Б}_2, \\ \text{Б}_\theta = \sin\omega\cdot\ \text{Б}_1+\cos\omega\cdot\ \text{Б}_2, \end{cases} \tag{36}$$

$$K_1^* = \text{Re}\,(Ka^{i\varepsilon}), \quad K_2^* = \text{Im}\,(Ka^{i\varepsilon}). \tag{37}$$

The stress intensity factor K has the form $K = |K|\,e^{i\psi}L^{-i\varepsilon}$. Therefore we have:

$$K_1^* = |K|\cos\psi_*, \quad K_2^* = |K|\sin\psi_*,$$

$$\psi_* = \psi+\varepsilon\text{L}n\frac{a}{L}. \tag{38}$$

† Assume Re $\mathbf{B}_{12} = 0$ for the sake of simplicity.

Substitute eqn (38) into eqn (34), we obtain:

$$\frac{G^{\text{kink}}}{G_i} = f^{(0)} + \eta f^{(1)} + \eta^2 f^{(2)}, \tag{39}$$

where

$$f^{(0)} = \frac{E_*}{2}\{B_{22}(e_{11}\cos\psi_* + e_{12}\sin\psi_*)^2 + B_{11}(e_{21}\cos\psi_* + e_{22}\sin\psi_*)^2\} \tag{40}$$

$$f^{(1)} = E_*\{B_{22}(e_{11}\cos\psi_* + e_{12}\sin\psi_*)\,\text{Б}_r + B_{11}(e_{21}\cos\psi_* + e_{22}\sin\psi_*)\,\text{Б}_\theta\} \tag{41}$$

$$f^{(2)} = \frac{E_*}{2}\{B_{22}\,\text{Б}_r^2 + B_{11}\,\text{Б}_\theta^2\}, \tag{42}$$

$$\eta = \sigma_0\sqrt{a}/|K| = \sigma_0\sqrt{a}/\sqrt{E_* G_i}. \tag{43}$$

The dimensionless parameter η is introduced by He *et al.* (1991) in the analysis of the kinking of a crack out of an interface for isotropic materials which characterizes the comprehensive effects of the stress σ_0 and the length of branch crack.

The functions $f^{(i)}$ depend on the load phase ψ_*, kink angle and the coefficients c_{ij}, Б_i, but are independent of the magnitude of K and the parameter η.

4. NUMERICAL RESULTS

The singular integral equations in eqn (18) can be represented in the dimensionless form. The numerical method proposed by Erdogan and his colleagues (1973) is used for solving eqn (18).

The singularity at the root of the branch is considered to be less than 1/2. This assumption seems to be consistent with the results of Bogy (1971) for the similar geometries of anisotropic media.

4.1. *Aligned orthotropic bimaterials*

Consider two dissimilar orthotropic materials bonded with the principal axes aligned. The interface is on the X axis and the crack is on the negative X axis. The components of **H** are:

$$\left\{\begin{aligned} H_{11} &= [2n\lambda^{1/4}\sqrt{s_{11}s_{22}}]_1 + [2n\lambda^{1/4}\sqrt{s_{11}s_{22}}]_2, \\ H_{22} &= [2n\lambda^{-1/4}\sqrt{s_{11}s_{22}}]_1 + [2n\lambda^{-1/4}\sqrt{s_{11}s_{22}}]_2, \\ H_{12} &= i\{[\sqrt{s_{11}s_{22}} + s_{12}]_1 - [\sqrt{s_{11}s_{22}} + s_{12}]_2\}, \\ \alpha &= (\Sigma - 1)/(\Sigma + 1), \quad \beta = iH_{12}/\sqrt{H_{11}H_{22}}, \\ \Sigma &= \frac{[s_{11}s_{12}]_2}{[s_{11}s_{22}]_1}. \end{aligned}\right. \tag{44}$$

Here $[\ ]_1$ designate quantities for material 1, and $[\ ]_2$ for material 2, s_{ij} are the elastic compliance tensors. The dimensionless parameters λ, n are given by:

$$\left\{\begin{aligned} &\lambda = \frac{s_{11}}{s_{22}} = \frac{E_2}{E_1}, \quad n = \sqrt{1+\rho}, \\ &\rho = (2s_{12} + s_{66})/2\sqrt{s_{11}s_{22}}. \end{aligned}\right. \tag{45}$$

The matrix **B** is:

$$\mathbf{B} = \begin{bmatrix} 2n\lambda^{1/4}\sqrt{s_{11}s_{22}} & i(\sqrt{s_{11}s_{22}}+s_{12}) \\ -i(\sqrt{s_{11}s_{22}}+s_{12}) & 2n\lambda^{-1/4}\sqrt{s_{11}s_{22}} \end{bmatrix}. \tag{46}$$

Eigenvector **W** takes the form:

$$\mathbf{W} = \frac{1}{2}\left[-i\sqrt{\frac{H_{22}}{H_{11}}}, 1\right]^T. \tag{47}$$

The traction on the interface can be described as:

$$\mathbf{t} = [\tau_{yx}, \sigma_y]^T = t_1\mathbf{W} + t_2\bar{\mathbf{W}}, \tag{48}$$

$$t_1 = \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}} = \sigma_y + i\sqrt{\frac{H_{11}}{H_{22}}}\tau_{xy}. \tag{49}$$

When $\varepsilon = 0$, we have:

$$\begin{cases} K_1 = \lim_{r\to 0}\sqrt{2\pi r}\sigma_y \\ K_2 = \lim_{r\to 0}\sqrt{2\pi r}\tau_{xy}\sqrt{\dfrac{H_{11}}{H_{22}}}. \end{cases} \tag{50}$$

It is worth noting the complex stress intensity factor K defined here in general is different from the conventional definition. For the aligned orthotropic bimaterials, when and only when the parameter ε equals zero and parameter H_{11}/H_{22} is equal to unity, the stress intensity factors K_1 and K_2, defined here is coincident with the conventional stress intensity factors.

For orthotropic materials, the two parameters λ and ρ measure the anisotropy. If both parameters λ and ρ approach unity, the material becomes isotropic.

In order to check our program, two sets of parameters are chosen: $\lambda_1 = \lambda_2 = 1$, $\rho_1 = 1.001$, $\rho_2 = 1.003$, $\alpha = \beta = 0$, $\psi_* = 45°$, $\eta = 0$ and $\eta = 0.25$.

It means that the materials have very weak anisotropy and eventually they can be considered as isotropic materials.

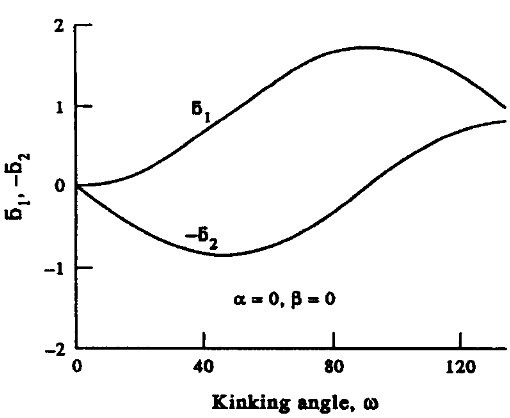

Fig. 3. Curve of $Б_1$ and $Б_2$ as a function of ω.

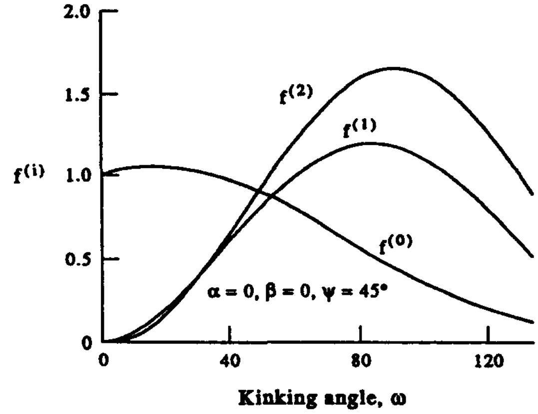

Fig. 4. Coefficients $f^{(i)}$ as a function of the kinking angle ω.

Our calculations result for G^{kink}/G_i, Б_1, Б_2 as the functions of kink angle ω agree within 1% with the results given by He *et al.* (1991).

In the following computation, the elastic constants for the two materials are taken to be $\lambda_1 = 1.0$, $\rho_1 = -0.19$, $\lambda_2 = 0.12$, $\rho_2 = 6.4$ which are corresponding to those of a typical Cu single crystal and the boron/epoxy composite.

The values of the coefficients c_{ij} are given by Wang *et al.* (1992) for six different combinations of α and β.

The values for the Б_1 and Б_2 as the functions of the kink angle ω are plotted in Fig. 3 for the case $\alpha = \beta = 0$. The coefficients $f^{(i)}$ vs ω are shown in Fig. 4 for the case $\alpha = \beta = 0$, $\psi = 45°$.

For the case $\alpha = \beta = 0$, $\psi = 45°$, the influence of σ_0 on the ratio G^{kink}/G_i is shown in Fig. 5. Here $G_s = G^{\text{kink}}$.

The ratio G^{kink}/G_i is increased when η increases. When $\eta = 0.5$, the ratio G^{kink}/G_i is increased by about 80%.

Figure 5 shows that there is a favorite σ_c, at which G^{kink} reaches maximum $G^{\text{kink}}_{\text{max}}$.

The ratio $G^{\text{kink}}_{\text{max}}/G_i$ as the function φ is plotted in Fig. 6 and Fig. 7 for the case $\alpha = \beta = 0$.

Figure 8 and Fig. 9 indicate the effects of parameter α on the ratio $G^{\text{kink}}_{\text{max}}/G_i$. It can be clearly seen that the ratio $G^{\text{kink}}_{\text{max}}/G_i$ will increase when α increases.

4.2. *Bicrystals with tilt grain boundary*

Figure 10 shows a tilt grain boundary of an orthotropic crystal, i.e. the two grains are misoriented but otherwise identical. The principal material axis x_1 is tilted from x by angles

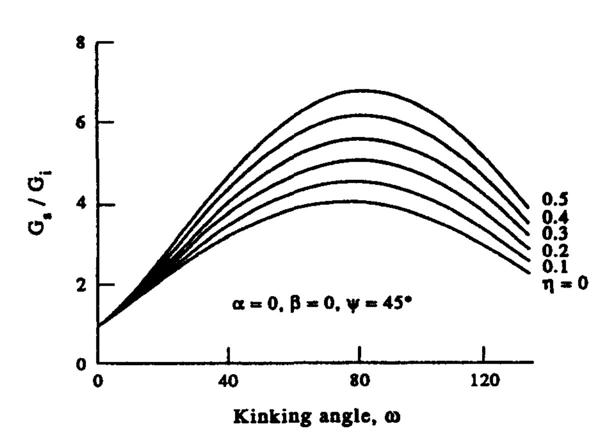

Fig. 5. Energy release rate ratio as a function of the kinking angle ω.

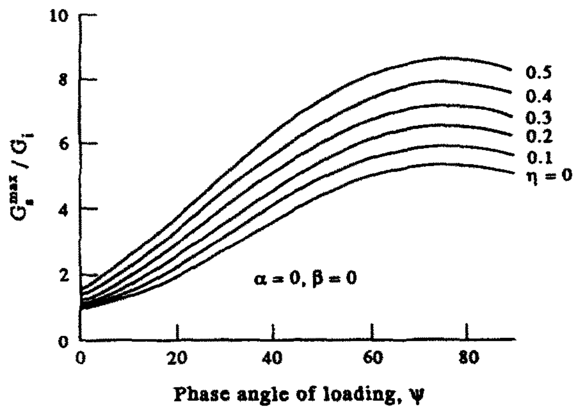

Fig. 6. Ratio of the maximum energy release rate of kinked crack to interface energy release rate as a function of the loading phase.

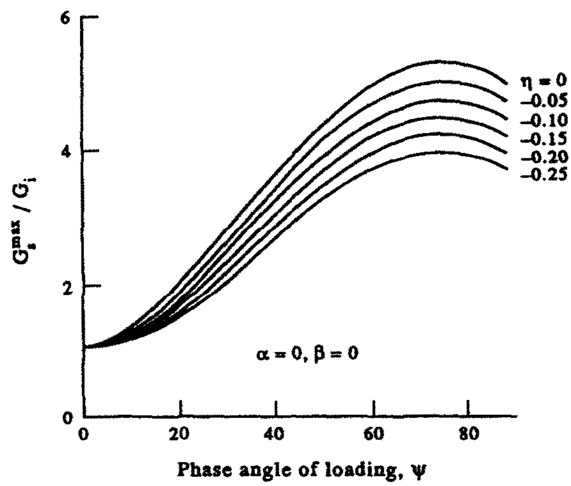

Fig. 7. Energy release rate vs the loading phase.

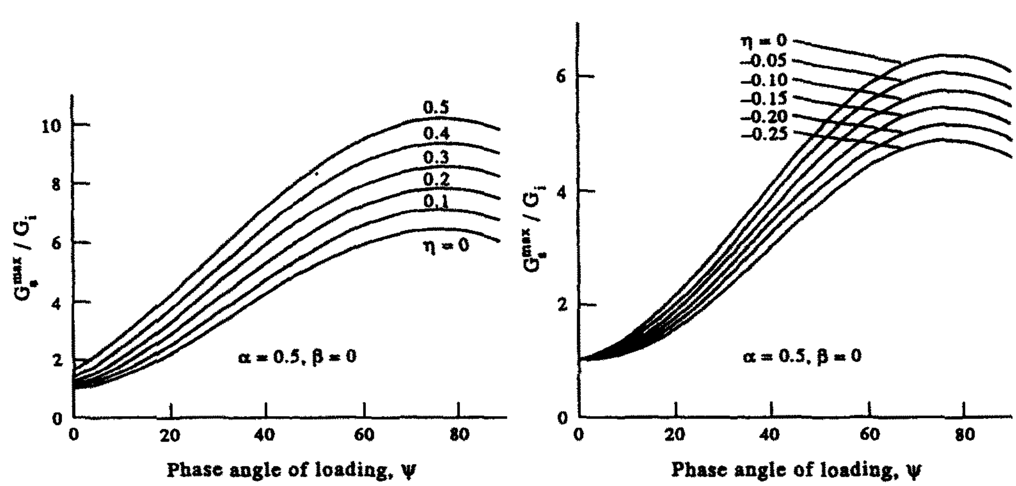

Fig. 8. Ratio of the maximum energy release rate of kinked crack to interface energy release rate as a function of the loading phase.

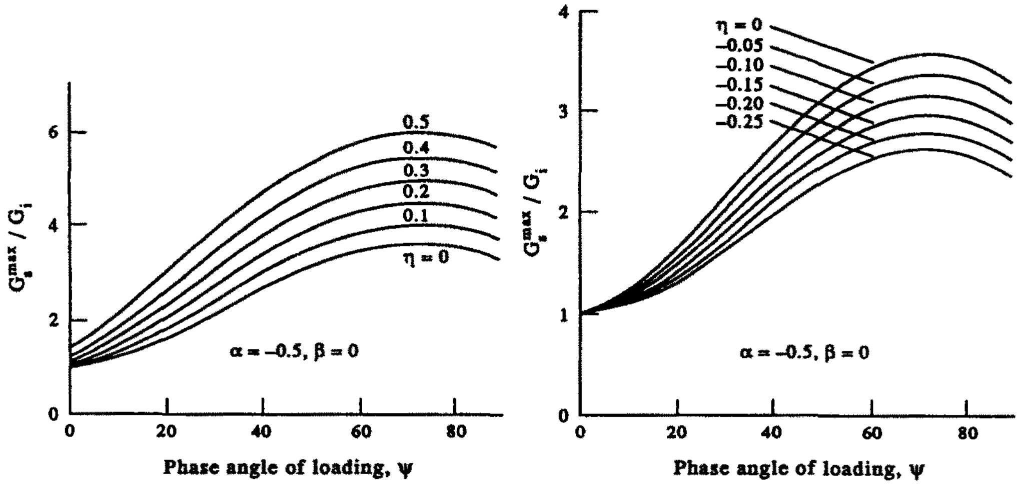

Fig. 9. Ratio of the maximum energy release rate of kinked crack to interface energy release rate as a function of the loading phase.

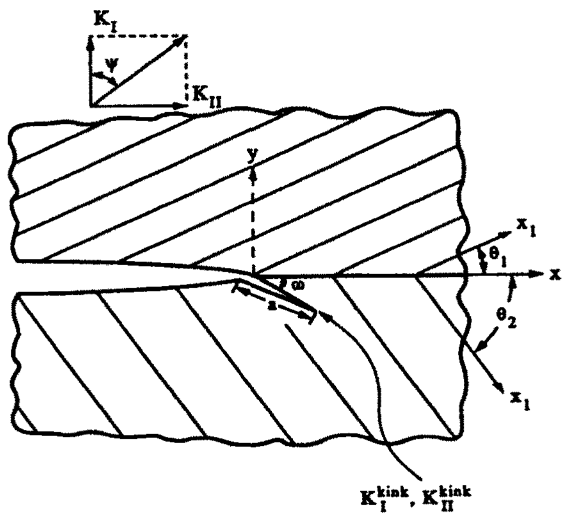

Fig. 10. A schematic of a small-scale kink problem. The principal direction x_1 is tilted from x-axis by θ_1 and θ_2, respectively.

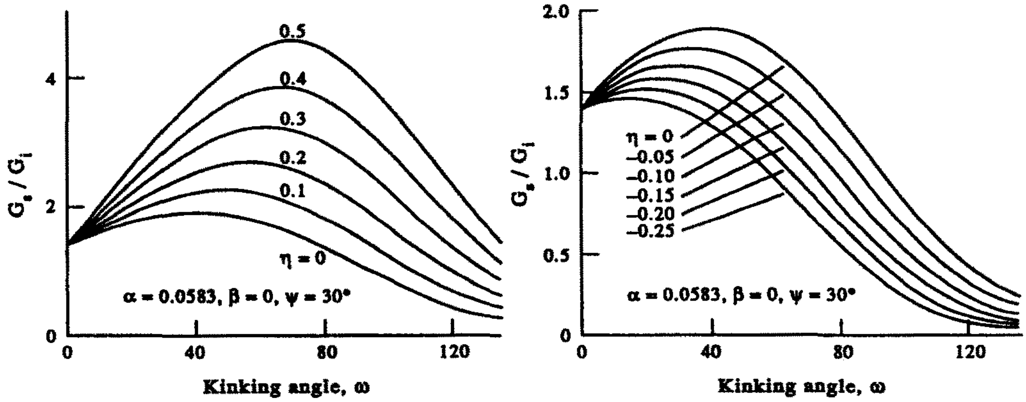

Fig. 11. Energy release rate ratio as a function of the kinking angle ω.

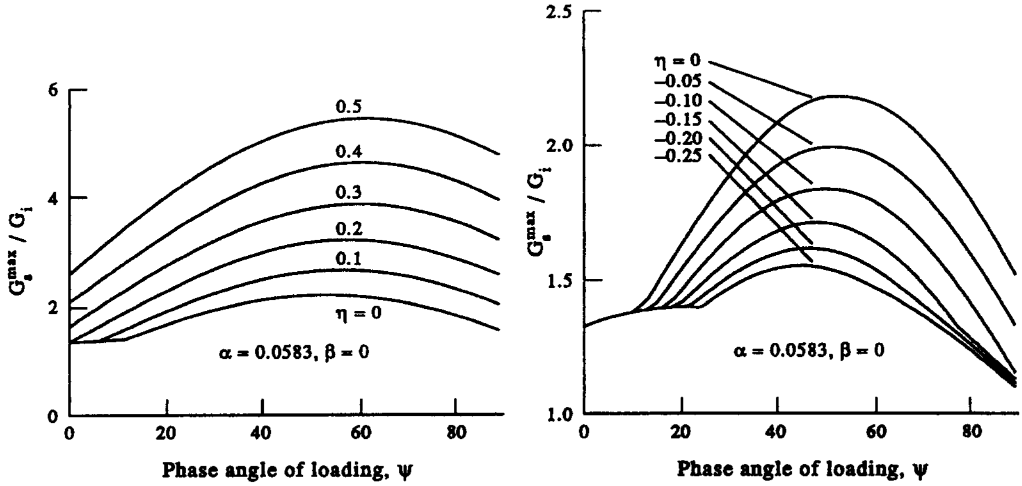

Fig. 12. Ratio of the maximum energy release rate of kinked crack to interface energy release rate as a function of the loading phase.

θ_1 and θ_2, respectively. The (x, y) plane is a plane of mirror symmetry, and the tilt axis and the crack front are normal to (x, y) plane. The matrix **H** for such a grain boundary crack is real (see Wang *et al.*, 1992).

Consider zinc single crystal (hexagonal system) at room temperature. The material properties for zinc in crystal axes are:

$$s_{11} = 8.0, \quad s_{22} = 28.2, \quad s_{66} = 25.0, \quad s_{12} = -6.1, \quad s_{13} = -0.5$$

with the unit $10^{-6}\ \mathrm{m^2\,MN^{-1}}$.

The corresponding parameters for plane strain are:

$$\lambda = s'_{11}/s'_{22} = 0.338$$

$$\rho = (2s'_{12}+s'_{66})/2\sqrt{s_{11}s'_{22}} = 0.439.$$

The calculation was carried out for the case $\theta_1 = 30°$, $\theta_2 = 75°$.

The calculated values of parameters α and β are $\alpha = 0.0583$, $\beta = 0.0$.

The values of the coefficients c_{ij} have been given by Wang *et al.* (1992).

Figures 11 and 12 show the influences of the stress σ_0 on the ratio $G_{\max}^{\text{kink}}/G_i$.

5. CONCLUSION AND DISCUSSION

A thorough analysis of kinking of a crack out of an interface between two dissimilar anisotropic elastic solids is presented in this paper. The coupled singular integral equations for distributed dislocation along the branch crack is obtained.

Explicit formulas of the stress intensity factors and the energy release rate for the branch crack are developed for aligned orthotropic bimaterials and misoriented orthotropic bicrystals.

The role of the stress parallel to the interface, σ_0, is emphasized. Inplane stress components can have a significant influence on the behavior of interface cracks. In particular, the tensile stress σ_0 will enhance the energy release rate G^{kink} for the branch crack, and causes the interface crack to depart from the interface. On the other hand, compressive stress σ_0 will reduce the G^{kink} and deactivate flaws around the interface.

The interface crack can advance along the interface or kinking out of the interface into the substrate. Broadly speaking, the tough substrate will result in the crack to extend along the interface. Kinking is favored if:

$$\frac{G^{\text{kink}}}{G_i} > \frac{\Gamma_s}{\Gamma_i}. \tag{51}$$

Here the Γ_s and Γ_i are the fracture toughness of the substrate and interface respectively.

For anisotropic substrate material, the toughness Γ_s depends also on the kink angle ω. Therefore the condition (51) can be rewritten as:

$$\Gamma_s(\omega) < \Gamma_i(\psi)G^{\text{kink}}(\omega, \psi)/G_i. \tag{52}$$

This paper provides the computation method for the ratio G^{kink}/G_i. The fracture resistance ratio $\Gamma_s(\omega)/\Gamma_i(\psi)$ should be quantified by the interfacial fracture testing and the fracture testing for the anisotropic substrate material.

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